Corner solution production function

A producer has the following production function $f(L,K) = max\{K, L\}$, is producing in the short run where $K = 25$ and L is the variable factor and sells its product at a market price $p = 10$. Answer:

1. What is the marginal productivity of labor when $L > 25$? And what is it when $L < 25$?

2. Find the interval of (positive) values for w that generate a positive unconditional labor demand.

Solution

1. With $K = 25$ we have: $f(L) = max\{25, L\}$, therefore an infinitesimal increase in L when positioned at $L = 3$ does not generate any production increase since the function will take the maximum between the two values and this maximum will be 25, hence the marginal productivity is 0. On the other hand, if we have $L = 50$, an increase in L will lead to positive marginal productivity, since the maximum will take the value of L as it is greater than 25.

2. The function to maximize is the profit:

$$B = 10max\{25, L\} - wL - r25$$

First, we should notice that the firm will demand either only L or only K because if $K \neq L$ then there would be wastage since the production function only takes the highest value between the two. Let's see what happens if the firm demands only K:

$$B = 10 * 25 - r25 = 250 - 25r$$

Now let's see what happens if it demands only L (and $L > 25$):

$$B = 10 * L - 25r - wL$$

Therefore, for the firm to demand from L, the profits from demanding L should be greater than the profits from only demanding K:

$$250 - 25r \leq 10L - 25r - wL$$

$$250 \leq 10L - wL$$

$$w \leq 10 - 250/L$$

Let's see possible values, if $L = 25$:

$$w \leq 0$$

If $L = 50$:

$$w \leq 5$$

If $L = 999999999$ approximately:

$$w \leq 10$$

Therefore, for there to be a positive demand for L we must have $w \in [0, 10)$.